Guitars and String Vibrations

Using abstraction and math to create beautiful music

Name: _____ Period: _____

Frequency can be calculated from the length of a string with the following inverse variation equation:

$$f = \frac{v}{2L}$$

where f is the frequency, L is the length of the string (in meters), and v is called the "wave speed" of the string (in meters per second).

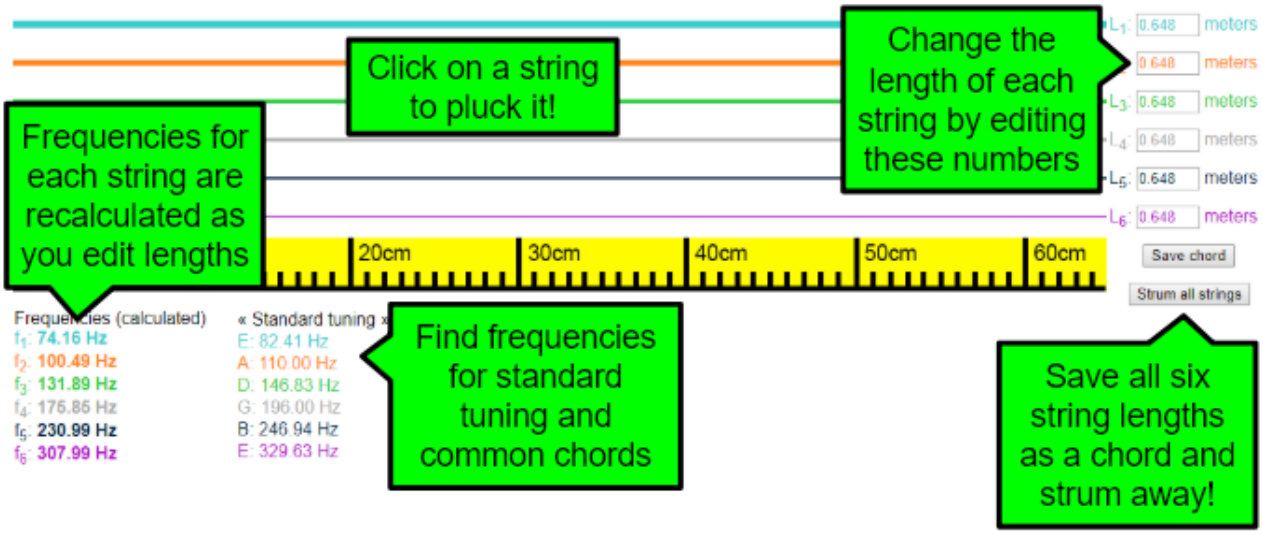
1. If a string's wave speed is 100m/s and it is 1m long, what frequency will it play when vibrating?

2. If a string's wave speed is 80m/s and it is 30cm long, what frequency will it play when vibrating?

3. If a string is 60cm long and it plays a 200Hz note when vibrating, what is that string's wave speed?

4. If a string's wave speed is 70m/s and we want it to play a C_4 note (131.87Hz), how long does the string need to be?

After you've answered the questions, load the digital guitar simulator at tinyurl.com/ctstrings



Tuning and Playing Your Digital Guitar

We need to calculate the wave speed for each string using a known length and frequency. Here's the first string as an example:

L_1: 0.648 meters $\quad$ f_1: **74.16 Hz**

$v_1 = f_1 \times 2\ L_1 = 74.16 \times 2 \times .648 =$ **96.11m/s**

Now, calculate the wave speed for the remaining five strings.

$v_2 = f_2 \times 2\ L_2 =$ _____

$v_3 = f_3 \times 2\ L_3 =$ _____

$v_4 = f_4 \times 2\ L_4 =$ _____

$v_5 = f_5 \times 2\ L_5 =$ _____

$v_6 = f_6 \times 2\ L_6 =$ _____

Once you have calculated the wave speed for each string, you can tune them to any notes you'd like! Calculate the string lengths for at least two more chords, making sure to save them within the simulation so we can play John Lennon's "Imagine" as a class!

For example, for "standard tuning", $f_1 = 82.41$ Hz, so $L_1 = v_1 / (2\ f_1) = 96.11 / (2 \times 82.41) =$ **.583m**

I chose _____ for my first chord.

$L_1 = v_1 / (2\ f_1) =$ _____

$L_2 = v_2 / (2\ f_2) =$ _____

$L_3 = v_3 / (2\ f_3) =$ _____

$L_4 = v_4 / (2\ f_4) =$ _____

$L_5 = v_5 / (2\ f_5) =$ _____

$L_6 = v_6 / (2\ f_6) =$ _____

I chose _____ for my second chord.

$L_1 = v_1 / (2\ f_1) =$ _____

$L_2 = v_2 / (2\ f_2) =$ _____

$L_3 = v_3 / (2\ f_3) =$ _____

$L_4 = v_4 / (2\ f_4) =$ _____

$L_5 = v_5 / (2\ f_5) =$ _____

$L_6 = v_6 / (2\ f_6) =$ _____

I chose _____ for my third (optional, if you have extra time) chord.

$L_1 = v_1 / (2\ f_1) =$ _____

$L_2 = v_2 / (2\ f_2) =$ _____

$L_3 = v_3 / (2\ f_3) =$ _____

$L_4 = v_4 / (2\ f_4) =$ _____

$L_5 = v_5 / (2\ f_5) =$ _____

$L_6 = v_6 / (2 f_6) =$ _____

“Imagine” by John Lennon

[Intro]
C F
C F

[Verse 1]
C F
Imagine there's no Heaven
C F
It's easy if you try
C F
No hell below us
C F
Above us only sky

[Chorus]
F Am Dm F
Imagine all the people
G C G
Living for today

[Verse 2]
C F
Imagine there's no countries
C F
It isn't hard to do
C F
Nothing to kill or die for
C F
And no religion too

[Chorus]
F Am Dm F
Imagine all the people
G C G
Living life in peace

[Bridge]
F G C E
You may say I'm a dreamer
F G C E
But I'm not the only one
F G C E
I hope someday you'll join us
F G C
And the world will be as one

[Verse 3]
C F
Imagine no possessions
C F
I wonder if you can
C F
No need for greed or hunger
C F
A brotherhood of man

[Chorus]
F Am Dm F
Imagine all the people
G C G
Sharing all the world

[Outro]
F G C E
You may say I'm a dreamer
F G C E
But I'm not the only one
F G C E
I hope someday you'll join us
F G C
And the world will be as one

Building a Cardboard and Rubber Band Ukulele

That's enough math for one day. Let's build! Follow the instructions below to create your ukulele.

1. Wrap one 12" rubber band, one 14" rubber band and one 17" rubber band around your box.
2. Insert two wooden dowels between the rubber bands and the closed, "bottom" part of your box. How do the different rubber bands sound?
3. Experiment with moving the dowels closer together and further apart. What happens to the notes of each rubber band as you move the dowels?
4. Place one of your dowels at one end of the box, and do not move it again. Using the reference notes in the digital guitar simulation to tune, mark the position of the second dowel to achieve different notes on each rubber band. This will help you play along with music - think of these markings like frets on a guitar!

Melodies to try on your new ukulele

Star Wars:
G_{low} D C B A G_{high} D C B A G_{high} D C B C A

Super Mario:
E E E C E G_{high} G_{low}

Twinkle Twinkle Little Star:
C C G G A A G F F E E D D C G G F F E E D G G F F E E D C C G G A A G F F E E D D C

Jingle Bells:
E E E E E E E G C D E F F F F F E E E E E D D E D G

Ode to Joy:
E E F G G F E D C C D E E D D E E F G G F E D C C D E D C C D D E C D E F E C D E F E D C D G_{low}

Note and frequency reference

Note	Frequency (Hz)	Note	Frequency (Hz)
C_2	65.41	G_3	196
$C^{\#}_2$	69.3	$G^{\#}_3$	207.65
D_2	73.42	A_3	220
$D^{\#}_2$	77.78	$A^{\#}_3$	233.08
E_2	82.41	B_3	246.94
F_2	87.31	C_4	261.63
$F^{\#}_2$	92.5	$C^{\#}_4$	277.18
G_2	98	D_4	293.66
$G^{\#}_2$	103.83	$D^{\#}_4$	311.13
A_2	110	E_4	329.63
$A^{\#}_2$	116.54	F_4	349.23
B_2	123.47	$F^{\#}_4$	369.99
C_3	130.81	G_4	392
$C^{\#}_3$	138.59	$G^{\#}_4$	415.3
D_3	146.83	A_4	440
$D^{\#}_3$	155.56	$A^{\#}_4$	466.16
E_3	164.81	B_4	493.88
F_3	174.61	C_5	523.25
$F^{\#}_3$	185		